INSTRUCTIONS:

1. Write your **Student Enrolment Number (SEN)** on the top right hand corner of this booklet.
2. This paper consists of **SIX QUESTIONS** and is out of 80 Skill Level.

<table>
<thead>
<tr>
<th>QUESTIONS</th>
<th>TOPICS</th>
<th>TOTAL SKILL LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE</td>
<td>ALGEBRA</td>
<td>17</td>
</tr>
<tr>
<td>TWO</td>
<td>CALCULUS</td>
<td>8</td>
</tr>
<tr>
<td>THREE</td>
<td>PROBABILITY</td>
<td>20</td>
</tr>
<tr>
<td>FOUR</td>
<td>SEQUENCE AND SERIES</td>
<td>9</td>
</tr>
<tr>
<td>FIVE</td>
<td>COMBINATIONS AND BINOMIAL THEOREM</td>
<td>14</td>
</tr>
<tr>
<td>SIX</td>
<td>TIME SERIES</td>
<td>12</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td><strong>80</strong></td>
</tr>
</tbody>
</table>

3. Answer ALL QUESTIONS. Write your answers in the spaces provided in this booklet.
4. Use a **BLUE** or **BLACK** ball point pen only for writing. Use a pencil for drawing if required.
5. If you need more space for answers, ask the Supervisor for extra paper. Write your **Student Enrolment Number (SEN)** on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.
6. A **4-page booklet** containing **Mathematical Formulae** and **table** is also provided.
7. Check that this booklet contains pages 2-19 in the correct order and that none of the pages is blank.

YOU MUST HAND IN THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.
QUESTION ONE  ALGEBRA

1. State a $2 \times 2$ system of equations that will give an infinitely many solutions.

$\begin{align*}
\frac{x}{2} &+ \frac{y}{2} = 1 \\
\frac{x}{2} &+ \frac{y}{2} = 1
\end{align*}$

2. On the Cartesian plane given below, sketch the graph of $y = x^{\frac{1}{2}}$, for $x \geq 0$

The curve shown in Figure 1 is a polynomial with root $\delta$. The Bisection Method is used to locate $\delta$ with a starting interval $[A, D]$.

3. Mark on the curve in Figure 1 the first two iterations of the Bisection method and labelled as $x_0$ and $x_1$.

Figure 1. Graph of $F(x)$
4. Mr. Flower loves to grow roses and lilies on his 30 hectares farm. His profit per hectare for lilies is $3500 and $4200 for roses. The farm is split into hectare paddocks with one type of flower per paddock. After making his planning decisions he has four constraints.

Let \( x = \) number of paddocks of roses

\( y = \) number of paddocks of lilies

i. State the equation for Mr. Flower’s Profit.

\[
\text{Profit} = 3500x + 4200y
\]

ii. Figure 2 shows the feasible region of the four constraints.

\[ x + y \leq 30; \ x \geq 5; \ 2x < y; \ y \geq 0 \]

iii. Determine the co-ordinates\((x, y)\) of corners B and C.
iv. Calculate the number of paddocks of roses and number of paddocks of lilies that will give the largest profit by completing the table given below.

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Co-ordinate (x,y)</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(5, 25)</td>
<td>$108500</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. **Table 1** gives the number \( y \) (in millions) of cell-phone subscribers from 1988 to 1997 where \( t \) is the number of years since 1987.

<table>
<thead>
<tr>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1.6</td>
<td>2.7</td>
<td>4.4</td>
<td>6.4</td>
<td>8.9</td>
<td>13.1</td>
<td>19.3</td>
<td>28.2</td>
<td>38.2</td>
<td>48.7</td>
</tr>
<tr>
<td>( \ln y )</td>
<td>0.47</td>
<td>0.99</td>
<td>1.48</td>
<td>1.86</td>
<td>2.19</td>
<td>2.57</td>
<td>2.96</td>
<td>3.34</td>
<td>3.64</td>
<td>3.89</td>
</tr>
</tbody>
</table>

**Figure 3** Plot of \( \ln y \) versus \( t \).
i. Referring to the graph in Figure 3, give a reason why \( y = a b^t \) is the best model for the number of cell-phone subscribers per year.

___________________________________________________________________
___________________________________________________________________
___________________________________________________________________
___________________________________________________________________

ii. Determine the equation \((y = a b^t)\) that model the number of cell-phone subscribers per year by calculating the value of \(a\) and \(b\).

___________________________________________________________________
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QUESTION TWO  

1. Differentiate \( x^2(x - 2)(x + 2) \)

2. Show that \( Y = Ae^{kt} \) where \( A \) is some constant, is a solution of the differential equation, \( \frac{dY}{dt} = kY \).
Dr. Papa tested his car fuel consumption at different constant speeds on a racetrack. After the tests he claims that fuel consumption, C is related to the speed of the car, v, by the formula

\[ C = 16 - 0.2v + \frac{v^2}{250} \]

3. Calculate the minimum fuel consumption of Dr. Papa’s car.
QUESTION THREE

PROBABILITY

A game of chance involves tossing a biased coin three times. Table 2 shows the probability distribution of random variable, X, the number of heads occurring. The expected value of X, E(X) is 1.2

<table>
<thead>
<tr>
<th>x</th>
<th>P(X = x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.216</td>
</tr>
<tr>
<td>1</td>
<td>0.432</td>
</tr>
<tr>
<td>2</td>
<td>0.288</td>
</tr>
<tr>
<td>3</td>
<td>0.064</td>
</tr>
</tbody>
</table>

1. Calculate the variance of X, VAR(X).

___________________________________________________________________

___________________________________________________________________

___________________________________________________________________

___________________________________________________________________

2. The Venn diagram in Figure 4 shows three events, A, B and C.

Figure 4
i. Name **TWO** (2) mutually exclusive events. Give your reason.

_____________________________________________________________
_____________________________________________________________
_____________________________________________________________

ii. Calculate $P(A \cup B \cap C')$

_____________________________________________________________
_____________________________________________________________
_____________________________________________________________
_____________________________________________________________

One sunny day there were 120 customers at Chef Zero restaurant. A quarter of its customers ask for water with their meal.

Four conditions must be satisfied for a particular situation to be modelled by a binomial distribution.

**Two** of the conditions are;

i. each trial is independent of others

ii. each trial has only two possible outcomes. (eg. each customer at Chef Zero restaurant may ask for water or does not ask for water with their meal)

3. State the other **TWO** (2) **conditions**.

(Use the customers at Chef Zero restaurant as an example.)

i. ____________________________________________________________

ii. ____________________________________________________________
3. Roger performs an experiment of selecting a marble from a bag and tossing a coin. The bag contains 3 red marbles and 5 green marbles. A marble is selected at random from the bag, its colour is noted and then the marble is returned to the bag.

- If a **red** marble is selected, a biased coin with probability \( \frac{2}{3} \) of landing a head is tossed.
- If a **green** marble is selected a fair coin is tossed.

i. Complete the tree diagram to show the possible outcomes and associated probabilities.

![Tree Diagram](image)

ii. Find the probability that Roger selects a red marble and obtains a head.
The number of laptops sold by an online shop follows a Poisson distribution at an average rate of 36 per year.

5. Calculate the probability that the shop sells \textbf{at least 1} laptop in a month period.

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{Skill level} & \textbf{3} \\
\hline
3 & \\
2 & \\
1 & \\
0 & \\
NR & \\
\hline
\end{tabular}
\end{center}

Random variable $X$ is normally distributed with a mean of 100 and a standard deviation of 15.

6. Calculate the value of $x$ such that $P(X < x) = 5$

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{Skill level} & \textbf{3} \\
\hline
3 & \\
2 & \\
1 & \\
0 & \\
NR & \\
\hline
\end{tabular}
\end{center}
To be eligible to apply for a scholarship to study for an MBBS, the applicant must have a distinction pass in Biology and also pass the TNFSC.

<table>
<thead>
<tr>
<th></th>
<th>Pass TNFSC</th>
<th>Does not pass TNFSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass Biology with distinction</td>
<td>22</td>
<td>8</td>
</tr>
<tr>
<td>Does not pass Biology with distinction</td>
<td>48</td>
<td>72</td>
</tr>
</tbody>
</table>

Table 3 summarises the result of 150 Year 13 pupils of one high school.

Let $B$ be the event “**Distinction pass in Biology**”, and

$T$ be the event “**pass in the TNFSC**”

7. A pupil is chosen at random. Given that he/she passed the TNFSC, calculate the probability that he/she passed Biology with Distinction.
QUESTION FOUR

SEQUENCE AND SERIES

1. Write down the first four terms of the exponential series $e^{-2x}$ in its simplest form

___________________________________________________________________
___________________________________________________________________
___________________________________________________________________

Mr. Brick designs the layout of a patio in front of his house. The first three layers of the patio are shown in Figure 5 and he needs 225 bricks to complete the patio.

2. Calculate the total number of layers for Mr. Brick’s patio.
A tennis ball is dropped from a height of 100 cm as shown in Figure 6. Each time the ball bounces, it loses 20% of the height from the previous height.

Figure 6

<80, 64, \( \frac{256}{5} \) (or 51.2), \ldots \ldots \ldots \rangle \text{ is the sequence for the bounces of the ball.}

3. Calculate the total distance the ball travels before it comes to rest on the ground.

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Skill level 4

<table>
<thead>
<tr>
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</thead>
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<tr>
<td>1</td>
</tr>
<tr>
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</tr>
<tr>
<td>NR</td>
</tr>
</tbody>
</table>
**QUESTION FIVE**  
**COMBINATION AND BINOMIAL THEOREM**

1. Use the formula for Permutation to evaluate $^{15}P_5$

   \[ \frac{15!}{(15-5)!} = \frac{15!}{10!} = \frac{15 \times 14 \times 13 \times 12 \times 11}{1 \times 2 \times 3 \times 4 \times 5} \]

   How many different license plates can be made by using either 3 digits or 3 letters assuming that you can use only odd digits and vowels? Repetition is not allowed.

2. Is this a combination or a permutation problem? Justify your answer.

   - Combination
   - Permutation

3. Determine the coefficient of $x^3y^3$ in the expansion of $(2x - y)^6$.

   \[ \binom{6}{3} \times (2x)^3 \times (-y)^3 = 20 \times 8x^3 \times (-1)^3 y^3 = -160x^3y^3 \]

4. Simplify $^{n+1}C_n + \binom{n}{n-1}$.

   (Use $\binom{n}{r} = \frac{n!}{r!(n-r)!}$).

   \[ \binom{n+1}{n} + \binom{n}{n-1} = \frac{(n+1)!}{n!(1)!} + \frac{n!}{(n-1)!(1)!} = n + 1 + 1 = n + 2 \]

---

**Skill Level 1**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td></td>
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</table>

**Skill Level 2**

<p>| | |</p>
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<tr>
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<td></td>
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<tr>
<td>1</td>
<td></td>
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<tr>
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<td></td>
</tr>
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<td></td>
</tr>
</tbody>
</table>

**Skill Level 3**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>NR</td>
<td></td>
</tr>
</tbody>
</table>
A group of four women and three men line up to be photographed.

5. Calculate the number of distinct ways they can line up if the shortest man stands on one end and three of the women want to stand together.

A committee of 7 has to be formed from 9 boys and 4 girls.

6. Calculate the different number of ways that the committee consists of at most 2 girls.
1. The plot in Figure 7 shows Australian monthly electricity production in the years 1955 to 1995. Mr. Analyst claims that the time series contains a seasonality component but not a cyclical component.

Figure 7

i. Justify Mr. Analyst’s claim.

ii. Identify another component shown by the plot in Figure 7.

iii. Describe the component named in ii.
2. The data shown in Table 4 represents the percentage part-time unemployment in Tonga. This is originally plotted in Figure 8.

Table 4: Part-Time Unemployment Rates with 4-points smoothing.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Percentage of Part-Time Unemployment Rates</th>
<th>Moving Mean (order 4)</th>
<th>Centered Moving mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992 Mar</td>
<td>9.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jun</td>
<td>9.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep</td>
<td>8.3</td>
<td>9.10</td>
<td>9.10</td>
</tr>
<tr>
<td>Dec</td>
<td>8.7</td>
<td>8.93</td>
<td>9.01</td>
</tr>
<tr>
<td>1993 Mar</td>
<td>9.9</td>
<td>8.60</td>
<td>8.76</td>
</tr>
<tr>
<td>Jun</td>
<td>8.8</td>
<td>8.40</td>
<td>**</td>
</tr>
<tr>
<td>Sep</td>
<td>7.0</td>
<td>8.25</td>
<td>8.33</td>
</tr>
<tr>
<td>Dec</td>
<td>7.9</td>
<td>*</td>
<td>8.09</td>
</tr>
<tr>
<td>1994 Mar</td>
<td>9.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jun</td>
<td>7.5</td>
<td>7.90</td>
<td>7.78</td>
</tr>
<tr>
<td>Sep</td>
<td>6.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec</td>
<td>6.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

i. Give a reason for using centered moving mean.
ii. Complete Table 4 by calculating the missing values marked by * and ** and plot the smoothing trend into the graph in Figure 8.

<table>
<thead>
<tr>
<th>Skill level 3</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>NR</td>
</tr>
<tr>
<td>1993</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>NR</td>
</tr>
<tr>
<td>1994</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>NR</td>
</tr>
</tbody>
</table>

Figure 8

iii. Seasonally adjust the percentage of part time unemployment in September for the year 1994.