INSTRUCTIONS

1. Write your Student Enrolment Number (SEN) on the top right hand corner of this booklet.
2. Answer ALL QUESTIONS. Write your answers in the spaces provided in this booklet.
3. If you need more space for answers, ask the Supervisor for extra paper. Write your SEN on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.
4. A 4-page booklet containing Mathematical Formulae and table is also provided.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>Topics</th>
<th>Pages</th>
<th>Time (mins)</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>ONE</td>
<td>ALGEBRA</td>
<td>2-5</td>
<td>40.5 mins</td>
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<tr>
<td>TWO</td>
<td>CALCULUS</td>
<td>6-7</td>
<td>20.25 mins</td>
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<tr>
<td>THREE</td>
<td>SEQUENCE &amp; SERIES</td>
<td>8-9</td>
<td>18 mins</td>
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<td>FOUR</td>
<td>PROBABILITY</td>
<td>10-14</td>
<td>36 mins</td>
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<td>FIVE</td>
<td>COMBINATIONS &amp; BINOMIAL THEOREM</td>
<td>15-18</td>
<td>33.75 mins</td>
<td>15%</td>
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<td>SIX</td>
<td>TME SERIES</td>
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<td>31.5 mins</td>
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<td>23</td>
<td>180 mins</td>
<td>80% marks</td>
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Check that this booklet contains pages 2-23 in the correct order and that none of the page is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.
QUESTION ONE: ALGEBRA

1. Use the Newton-Raphson method is used to find the root of 
   \( f(x) = x^4 - 2x^3 + 2x - 2 \) with \( x_1 = 1.73764 \). (i.e. three iterations) 
   (hint: round your calculations to 5 decimal places)
2. One of the courier company here in Tonga express their delivery fees depend on the size of the packages. Small packages cost $5, medium packages cost $16 and the large packages cost $22. In one day the company receives $282 for 22 packages altogether. There were as many medium packages as the sum of the numbers of small and large packages. Form a system of equations and solve to determine the number of packages of each size.
3. Justify ONE (1) disadvantage of using Newton-Raphson when comparing to the Bisection Method.

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4. Sketch the graph of \( f(x) = x^{\frac{3}{5}}, -5 < x < 5 \) on the grid below.
5. “A motor cycle dealer has 250 bikes on his lot. There is a mixture of $x$ off-road bikes and $y$ on-road bikes. There are at least twice as many on-road bikes as there are off-road bikes. Write the equations that reflect the given information and solve it.

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6. A tank containing 5050ml loses water at a rate of 60ml per 35 seconds. At that time the leak is plugged and stays plugged for 20 seconds till the plug comes out and the rest of the water pours out at the rate of 70ml a second till the tank is empty. Find the volume of the water when $t = 1$ min.
QUESTION TWO: CALCULUS

1. What is the derivative of \( y = \frac{-3e^{-2x}}{4} + 2x^3 \)

2. If \( \frac{dy}{dx} \propto y \), solve the differential equation given
   
   - \( y = 6 \) when \( x = 2 \)
   - \( y = 20 \) when \( x = 5 \)

3. A textile manufacturer makes blankets, and sells them for $25 each. The cost in dollars, of making \( x \) blankets is given by the function: \( C(x) = 5000 + 4x + 0.01x^2 \). Find the number of blankets that the manufacturer should produce in order to maximise the profit.
4. Differentiate $e^{2x}(3x^2 + 4)$

Skill level

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QUESTION THREE: SEQUENCE & SERIES

1. Write down the first four terms in the series of $e^{2x}(2 - x)$.

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2. Show that $T_n = -180 \left(\frac{-1}{2^n}\right)$ is a geometric sequence.

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Skill level 1

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3. Insert nine numbers between 3 and 21 so that all eleven numbers are in arithmetic sequence.

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4. Find k given that 4, k and \( k^2 - 1 \) are consecutive geometric terms.

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QUESTION FOUR: PROBABILITY

1. A pack of cards has four suits and each has 13 cards (4 picture cards, 9 number cards). A person is dealt two cards. Construct a probability tree that the two cards are the same number.

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2. The probability a student comes to a mathematics class with a pen is 0.96, with a mathematic book is 0.93 and comes with pen or mathematics book is 0.98.

   a. Use this information to draw a Venn diagram to identify the Probability of those that came to class with only one item.

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b. **Calculate the probability of those who came to class with only one item (either a pen or mathematics book)**

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3. **Two defective fuses have been mixed up with three good ones. The fuses are tested, one by ones, until both defective ones are found. What is the probability that the testing of exactly three fuses is required?**

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</table>
4. From a squad of 13 which includes 4 brothers, a team of 7 is randomly selected by drawing names from a hat. Determine the probability that the team contains at least 2 of the brothers.
5. In a multiple choice test there are 10 questions and each question has 5 choices, one of which is correct. If 70% is the pass mark and Timote (who know nothing) guesses at each answer, determine the probability that he will pass.
6. When customers of Star Bank sign up for their credit card, they were asked a question later “did you read the contract carefully? The findings were that 44% read every word, 33% read enough to understand the contract, 11% just glance at it, and 4% don’t read it at all. For a sample of 300 people, what is the probability that at most 210 did spend some time to read the contract?
QUESTION FIVE: COMBINATION & BINOMIAL THEOREM

1. Simplify $\frac{(n+2)!+(n+1)!}{(n+3)}$

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2. Evaluate $^{10}C_2$

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3. A television news director wishes to use three news stories on an evening show. One story will be the lead story, one will be the second story, and the last will be a closing story. If the director has a total of 10 stories to choose from, how many possible ways can the program be set up?

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4. Use factorial form to simply \( \binom{10}{4} + \binom{10}{3} \). Give your answer as a Binomial coefficient

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5. The letters of CALCULUS are all used to form permutations. How many permutations start with A and end S?

Skill level 2

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6. Use the Pascal Triangle to expand \((x + 2)^7\) to its simplest form.

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7. By finding \(T_{n+1} = \binom{n}{r} a^{n-r} b^r\) in the simplest form, find the coefficient of \(x^{-4}\) in the expansion of \((2x + \frac{1}{x})^{10}\)

Skill level 2

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8. How many different ways can five students (A, B, C, D, E) sit in a row at the school assembly if C must sit to the right of, but not necessarily next to, B.
1. Study the graph and comment on the nature of the seasonal variation whether there are exceptions to this.
2. This table shows time series for the number of absentees from one of the High School at Nonu Island over a four-week period.

<table>
<thead>
<tr>
<th>Day of week</th>
<th>Number of absentees</th>
<th>Moving Mean (order 5)</th>
<th>Individual seasonal effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon</td>
<td>30</td>
<td></td>
<td></td>
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<tr>
<td>Tues</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wed</td>
<td>53</td>
<td>34.4</td>
<td>18.6</td>
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<tr>
<td>Thurs</td>
<td>20</td>
<td>39</td>
<td>-19</td>
</tr>
<tr>
<td>Fri</td>
<td>23</td>
<td>36.4</td>
<td>-13.4</td>
</tr>
<tr>
<td>Mon</td>
<td>53</td>
<td>35.2</td>
<td>17.8</td>
</tr>
<tr>
<td>Tues</td>
<td>33</td>
<td>39.2</td>
<td>-6.2</td>
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<tr>
<td>Wed</td>
<td>47</td>
<td>46.6</td>
<td>0.4</td>
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<tr>
<td>Thurs</td>
<td>40</td>
<td>48.8</td>
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<td>Fri</td>
<td>60</td>
<td>46.6</td>
<td>13.4</td>
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<tr>
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<td>31.2</td>
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<td>28.6</td>
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<td>Fri</td>
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</tbody>
</table>

a. Explain why moving means of order 5 are used here to smooth the data.
b. Calculate the missing moving entries and the seasonal effects.
3. a. The individual seasonal effects for Monday are always positive. Explain what this means for this High School attendance.
b. Apply smoothing values to the graph above to describe the Trend.