Tonga National Form Seven Certificate

Mathematics with Statistics

2014

QUESTION and ANSWER BOOKLET

Time allowed: Three hours

INSTRUCTIONS

Write your Student Personal Identification Number (SPIN) on the top right corner of this page and on the fold-out flap on the last page.

Write your Student Personal Identification Number (SPIN) on the right corner of this page and on the foldout flap on the last page.

Unless otherwise stated, numerical answers correct to four significant figures will be adequate.

If you need more spaces for answer, ask the Supervisor for extra paper. Write your Spin on all extra sheets used and clearly number the questions. Attach the extra sheets at the appropriate places in this booklet.

A 4-page booklet containing mathematical formulae and table is also provided.

SECTION A

You must answer QUESTION ONE

35 Marks

65 mins

SECTION B

Answer only FIVE QUESTIONS

From Question TWO to SEVEN.

Each Question is worth 13 Marks

65 Marks

117 mins

100 MARKS

Check that this booklet contain 18 in the correct order and that none of these page are blank. YOU MUST HAND IN THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.
SECTION A: COMPULSORY

QUESTION ONE: COMPULSORY (35 MARKS)

Show all necessary working in each part.

a) Suppose that we have a sample space with five equally likely experimental outcomes: \(E_1, E_2, E_3, E_4\) and \(E_5\).

Let 
\[
A = \{E_1, E_2\} \\
B = \{E_3, E_4\} \\
C = \{E_2, E_3, E_5\}
\]

1. Find \(P(A')\) \hspace{1cm} (1 mark)

2. Find \(P(B \cap C')\) \hspace{1cm} (2 marks)

b) 1. Expand and simplify \(nC_2\). Show all working. \hspace{1cm} (2 marks)

2. Solve \(nC_2 = 6\) \hspace{1cm} (1 mark)
c) Find the coefficient of $x^{12}$ in the expansion of $(1 + 2x^3)^{11}$ (2 marks)

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(d) Given that

<table>
<thead>
<tr>
<th></th>
<th>$\sum fx^2$</th>
<th>$\sum fx$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6978</td>
<td>490</td>
<td>35</td>
</tr>
<tr>
<td>B</td>
<td>6468</td>
<td>174</td>
<td>34</td>
</tr>
</tbody>
</table>

Use the information given on the table to calculate the:

1) Mean of $A$ (1 mark)

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2) Standard deviation of $B$ (2 marks)

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e) The value of a diamond rock over a period of time can be modelled by this equation $P = 5800 e^{0.25t}$ where $t$ is the number of years since purchase and $P$ is the value in dollars.

1) Write $P$ in form of $P = am^x$. (1 mark)

2) What is the percentage rate of increase in value? (1 mark)

f) In a consignment of farm, apple that a supermarket receives each day there are, on average 6 rotten apples. Use the Poisson distribution to calculate the probability that on a randomly chosen day there are more than 4 rotten apples. (2 marks)

g) How many different 7 digits telephone numbers are possible if the first two digit cannot be zero and no digit may repeat? (2 marks)
h) The probability that a person will pass a math exam is $\frac{5}{8}$ and the probability that his friend will pass is $\frac{6}{7}$.
Assuming the events are independent what is the probability that at least one will pass? (2 marks)

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i) Evaluate this series $456 - 228 + 114 - 57 + \ldots$ (2 marks)

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j) A biased six-sided die has the following probabilities. Calculate the probability if the die tossed twice and have the same faces. (2 marks)

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
<td>0.03</td>
<td>0.13</td>
<td>0.51</td>
<td>0.06</td>
<td>0.12</td>
<td>0.15</td>
</tr>
</tbody>
</table>
k) Sketch the graph of the function below. No need to use graph paper or grid but clearly show scale on the axes. (3 marks)

\[ f(x) = \begin{cases} 
  x, & x \geq 3 \\
  1, & -2 \leq x < 3 \\
  x + 1, & x < -2 
\end{cases} \]

l) Function \( C \) is given by \( x^4 - 4x - 8 \).

1. Use calculus to calculate the co-ordinate of the vertex of \( C \) (3 marks)

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2. Show that there is a root of function \( C \) in the interval of \([-2, -1]\) 

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m) An arithmetic sequence is defined by \( A_i = 3i - 2 \).

1. Calculate the first 4 terms 

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2. Show that the sum of the first \( n \) terms of the sequence is given by

\[
S_n = \frac{3n^2 - n}{2}
\] 

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QUESTION TWO: 

A) Three warplanes bomb a target in succession. The probability of each correctly scoring a hit is 0.5, 0.4 and 0.6 respectively. The second will bomb only if the first misses the target, if they both missed the target than the third plane will take it chance. Find the probability that

1) target is hit

2) All fails to score hits.

B) For the events A and B,

\[ P(A \cap B') = 0.32 \, , \, P(A' \cap B) = 0.11 \, \text{and} \, P(A \cup B)' = 0.35 \]

1) Draw a Venn diagram to illustrate the complete sample space for the events A and B
2) Calculate the \( P(A) \) (1 mark)

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3) Determine whether or not A and B are independent (2 marks)

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C) Solve this system of equations. (4 marks)

\[
\begin{align*}
\ x + y - z &= 11, \\
8x + 3y - 6z &= 1 \\
-4x - y + 3z &= 1
\end{align*}
\]
QUESTION THREE:
A) The cubic \( f(x) = x^3 - 4x \) and \( g(x) = x^2 - 3x + 4 \) intersect once

1) Form an appropriate function \( h(x) \), and its derivative to find the point of intersection. 
(2 marks)

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2) Use the Newton-Raphson method to find the \( x \) ordinate of intersection, using \( x = 2.857 \) as the initial value
(3 marks)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x_n )</th>
<th>( h(x_n) )</th>
<th>( h'(x_n) )</th>
<th>( x_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.857</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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B) The random variable X has probability distribution:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X=x)</td>
<td>0.10</td>
<td>p</td>
<td>0.20</td>
<td>q</td>
<td>0.30</td>
</tr>
</tbody>
</table>

1) Given that \( E(X) = 3.5 \), write down two equations involving \( p \) and \( q \) (your answer should be in the most simplified form) (2 marks)

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2) Find \( p \) and \( q \) (2 marks)

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3) Calculate \( \text{Var}(X) \) (2 marks)

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4) Calculate \(\text{Var}(3 - 4X)\)  

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QUESTION FOUR: (13 MARKS)

A) The height in metres of the vertical rods of an early suspension bridge, as you move out from the centre, form the sequence, 1.1, 1.4, 1.9, 2.6, …

1) What are the likely heights of the 5th and 6th rods in this sequence (2 marks)

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2) The height, H metres, of the nth rod in the sequence is given by the formula \( H = A + Bn^2 \). Find the values of A and B and write down the formula. (2 marks)

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B) An initial population of 750 endangered turtles triples each year for 5 years.

1) Find the growth factor for the population. (1 mark)

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2.) Write the mathematical model involving the growth of turtles (2 marks)

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3.) Calculate the population after 5 years. (2 marks)

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C) According to a candy company, packages of a certain candy contain 17% orange candies. Find the approximate probability that the random sample of 300 candies will contain 20% or more orange candies. [Hint: Using a normal approximation, what is the probability that at least 20% of 300 randomly sampled candies will be orange?] (4 marks)

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QUESTION FIVE: (13 MARKS)

A) Given that this piece-wise function is continuous, having an unknown value \( k \),

\[ f(x) = \begin{cases} 
  k\sqrt{x} + 1, & 0 \leq x \leq 3 \\
  5 - x, & 3 < x \leq 5 \\
  2x, & x > 5 
\end{cases} \]

1) Solve for \( k \) (show all working) (2 marks)

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2) Sketch the piece-wise function. (3 marks)
B) Employees of a large corporation are concerned about the declining quality of medical services provided by their group health insurance. A random sample of 100 office visits by employees of this corporation to primary care physicians during 2001 found that the doctors spent an average of 19 minutes with each patient with a standard deviation of 2.7 minutes. This year a random sample of 108 such visits showed that the doctors spent an average of 15.5 minutes with each patient with a standard deviation of 2.1 minutes.

1) Assuming 98% confidence confidence, calculate the margin of error for these two populations.

(2 marks)

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2) Construct a 95% confidence interval for the difference between the two population means for the two years.

(2 marks)

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C) Plot the following inequalities on the same axes and indicate the region that satisfies all the inequalities

\[ y - 2x \leq 4, \quad x \leq 2 \quad \text{and} \quad 2y + x \geq 4 \]

(4 marks)
QUESTION SIX: (13 MARKS)

A) Let \( f(x) = 2x + \frac{200}{x} \)

1) Explain why the graph of \( f(x) \) gets closer to a straight line when \( x \) gets larger in magnitude

(1 mark)

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2) Show that the graph of \( f(x) \) has two turning point. State, with reason, which is the maximum turning point.

(4 marks)

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B) State TWO advantage of using Newton-Raphson

(2 marks)

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C) A random variable $x$ represents the number of exams a randomly selected student has on any given day. The probability distribution function of $x$ is given by the following formula:

$$p(x) = \frac{x^2 - x + 2}{8} \quad \text{for } x = 0, 1, 2$$

1) Write the probability distribution table (1 mark)

<table>
<thead>
<tr>
<th>$x$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2) Find the probability that a student has at most one exam on a given day (2 marks)

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D) Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours. (3 marks)

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A) The data gives the tones of apples exported by Golden Apply Company from New Zealand during the five year period 2000 – 2004

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Tonnes of apples</th>
<th>Moving average</th>
<th>Centred Moving Average</th>
<th>Individual Seasonal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>34.85</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>33.65</td>
<td>35.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30.3</td>
<td>40.83</td>
<td>38.16</td>
<td>-7.26</td>
</tr>
<tr>
<td>4</td>
<td>42.55</td>
<td>45.80</td>
<td>43.32</td>
<td>-0.77</td>
</tr>
<tr>
<td>2001</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>56.21</td>
<td>50.68</td>
<td>48.24</td>
<td>7.97</td>
</tr>
<tr>
<td>2</td>
<td>53.55</td>
<td>53.39</td>
<td>52.04</td>
<td>1.52</td>
</tr>
<tr>
<td>3</td>
<td>50.41</td>
<td>61.01</td>
<td>57.20</td>
<td>-6.79</td>
</tr>
<tr>
<td>4</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>2002</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>73.06</td>
<td>74.19</td>
<td>74.19</td>
<td>4.87</td>
</tr>
<tr>
<td>2</td>
<td>74.45</td>
<td>76.28</td>
<td>75.48</td>
<td>-1.03</td>
</tr>
<tr>
<td>3</td>
<td>69.05</td>
<td>82.33</td>
<td>79.55</td>
<td>-10.50</td>
</tr>
<tr>
<td>4</td>
<td>84.55</td>
<td>87.40</td>
<td>84.87</td>
<td>-8.32</td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>101.26</td>
<td>91.56</td>
<td>89.66</td>
<td>11.58</td>
</tr>
<tr>
<td>2</td>
<td>94.75</td>
<td>97.04</td>
<td>94.50</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>87.26</td>
<td>102.52</td>
<td>93.78</td>
<td>-12.50</td>
</tr>
<tr>
<td>4</td>
<td>104.85</td>
<td>107.52</td>
<td>105.02</td>
<td>-0.17</td>
</tr>
<tr>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>123.2</td>
<td>112.31</td>
<td>103.92</td>
<td>13.20</td>
</tr>
<tr>
<td>2</td>
<td>114.75</td>
<td>117.02</td>
<td>114.67</td>
<td>0.88</td>
</tr>
<tr>
<td>3</td>
<td>106.45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>123.68</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1) Complete the missing values (4 marks)
2) Explain the purpose of smoothing the time series (1 mark)
3) Study the graphs and describe two features of the export done by Golden Apply Company during the five year period 2000 – 2004 (2 marks)

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4) Find the average seasonal effects for the first quarter (1 mark)

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5) Predict the value of tones exported in the First quarter of 2005. (2 marks)

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B) Use the identity of the combination to simplify \( ^{n+3}C_{n+1} \) to its simplest rational form. (2 marks)

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